Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2014

MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

MATHE1

This question paper consists of 13 pages including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TWELVE questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. Answers only will not necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

7. Number the answers correctly according to the numbering system used in this question paper.

8. Write neatly and legibly.

9. An information sheet with formulae is included at the end of the question paper.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 7x = 0$ (2)

1.1.2 $5x^2 = 3x + 6$ (4)

1.1.3 $3x^{\frac{2}{3}} - 13x^{\frac{1}{3}} - 10 = 0$ (3)

1.2 Given the expression: $2x^2 - 7x - 15$

1.2.1 Solve for x if $2x^2 - 7x - 15 \geq 0$ (4)

1.2.2 Hence or otherwise, determine for which positive values of x will the following expression be real:

$$\frac{\sqrt{2x^2 - 7x - 15}}{x - 8}$$

(2)
[15]

QUESTION 2

2.1 Solve for x and y simultaneously in the following equations:

$$2x^2 - 3xy = -4 \quad \text{and} \quad 4^{x+y} = 2^{y+4}$$

(7)

2.2 Discuss, without solving the equation, the nature of the roots for the equation:

$$3x^2 - 5x + 3 = 0$$

(2)
[9]

QUESTION 3

The first four terms of a quadratic sequence are as follows:

$$2 \ ; \ 10 \ ; \ 14 \ ; \ 14 \ ; \ \ldots$$

3.1 Give the value of the next term in the sequence. (1)

3.2 Determine the n-th term of the sequence. (4)
[5]

QUESTION 4

4.1 Given the following number pattern which is a combination of a linear and a geometric pattern:

$$3\ ;\ \frac{1}{2}\ ;\ 3\ ;\ \frac{4}{10}\ ;\ 3\ ;\ \frac{16}{50}\ ;\ \ldots$$

4.1.1 Write down the values of the next TWO terms of the pattern. (2)

4.1.2 Calculate the sum of the first thirty-five terms of the pattern. (5)

4.2 Calculate:

$$\sum_{n=3}^{\infty} 5.3^{1-n}$$

(4)

[11]

QUESTION 5

5.1 The sequence consisting of all natural numbers from 25 to 999 is given as follows:

$$25\ ;\ 26\ ;\ 27\ ;\ 28\ ;\ 29\ ;\ \ldots\ ;\ 999$$

5.1.1 Write down the first three even numbers in the given sequence. (1)

5.1.2 Determine the sum of all the even numbers in the given sequence. (5)

5.2 The m-th term of an arithmetic sequence is k and the k-th term of the same sequence is m. Determine the value of the common difference (d) if $m \neq k$. (4)

[10]

QUESTION 6

6.1 Tembi buys a new car for R245 000. The value of the car depreciates at 13% per annum according to the reducing-balance method.

After how many years will the value of the car be R83 543? (4)

6.2 Mr Williams buys a house for R450 000. He pays a deposit of 10% and takes out a bank loan for the balance.

6.2.1 Calculate the value of the loan. (2)

6.2.2 He pays back the loan by means of equal monthly instalments over a period of 20 years. The first payment is made one month after the allocation of the loan. Interest is calculated at 8% per annum, compounded monthly.

Calculate the value of the monthly instalment. (4)

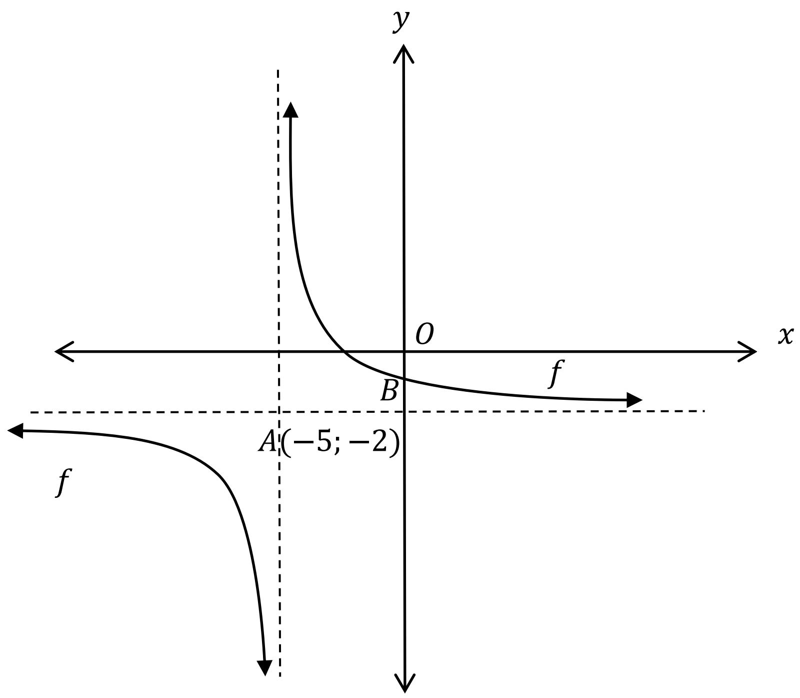
6.2.3 He decides to settle the loan after 17 years.

Calculate the outstanding balance on the loan if the last payment is made at the end of the 17^{th} year. (3)

[13]

QUESTION 7

7.1 The sketch shows the graph of $f(x) = \frac{2}{x+p} + q$. $A(-5;-2)$ is the point of intersection of the asymptotes of f. B is the y-intercept of f.



7.1.1 Write down the equation of f. (2)

7.1.2 Calculate the coordinates of B. (2)

7.1.3 Determine the equation of g, the straight line passing through A and B. (4)

7.1.4 Hence, determine the coordinates of the other point of intersection of f and g. (4)

7.2 Given $f(x) = \left(\frac{1}{5}\right)^x$

7.2.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

7.2.2 Write down the range of g if $g(x) = f(x) - 2$. (1)

7.2.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

7.2.4 Draw a neat sketch of f^{-1}. Clearly show all intercepts with the axes. (2)

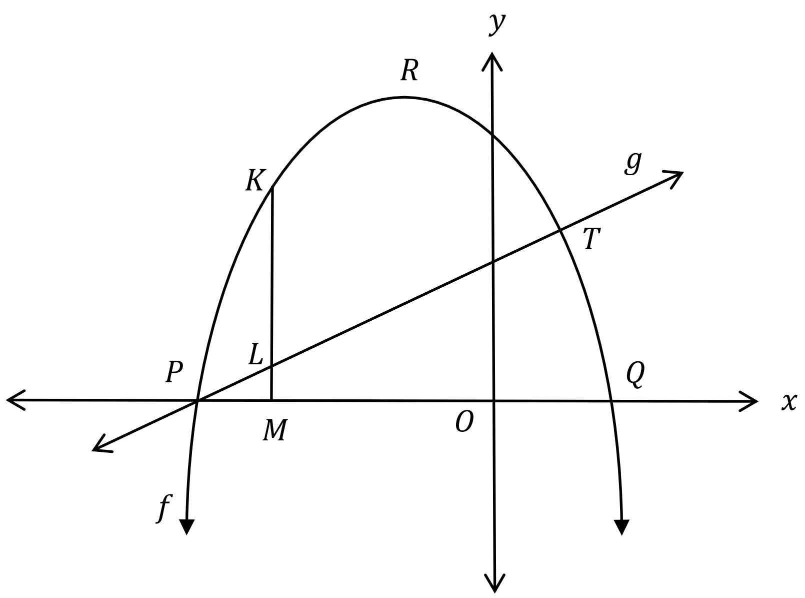
7.2.5 For which value(s) of x is $f^{-1}(x) \geq -1$? (2)

[21]

QUESTION 8

The sketch shows the graphs of the functions $f(x) = -x^2 - x + 12$ and $g(x) = x + 4$.

P and Q are the x-intercepts of f, while R is the turning point of f. The functions intersect at T and P. K is a point on f, L is a point on g and M lies on the x-axis such that KLM is a straight line parallel to the y-axis.



8.1 Determine the x-coordinates of P and Q. (3)

8.2 Determine the coordinates of R. (2)

8.3 Determine the coordinates of M if $KL = 6\frac{3}{4}$ units. (4)

8.4 Determine for which value(s) of x will:

$$x \, . f(x) > 0$$ (2)

8.5 For which real value(s) of k will $-x^2 - x + 12 = k$ have two negative, unequal roots? (2)

8.6 Give the equation of h in the form $h(x) = a(x + p)^2 + q$ if h is the reflection of f in the straight line $x = 1$. (2)

[15]

QUESTION 9

9.1 Given: $f(x) = -5x^2 + 2x$

Determine $f'(x)$ from first principles. (5)

9.2 Determine $\frac{dy}{dx}$ if $y = \frac{8}{x^4} + \sqrt[3]{x^2}$ (4)

-

9.3 Given: $f(x) = -x^3 + 3x - 2$

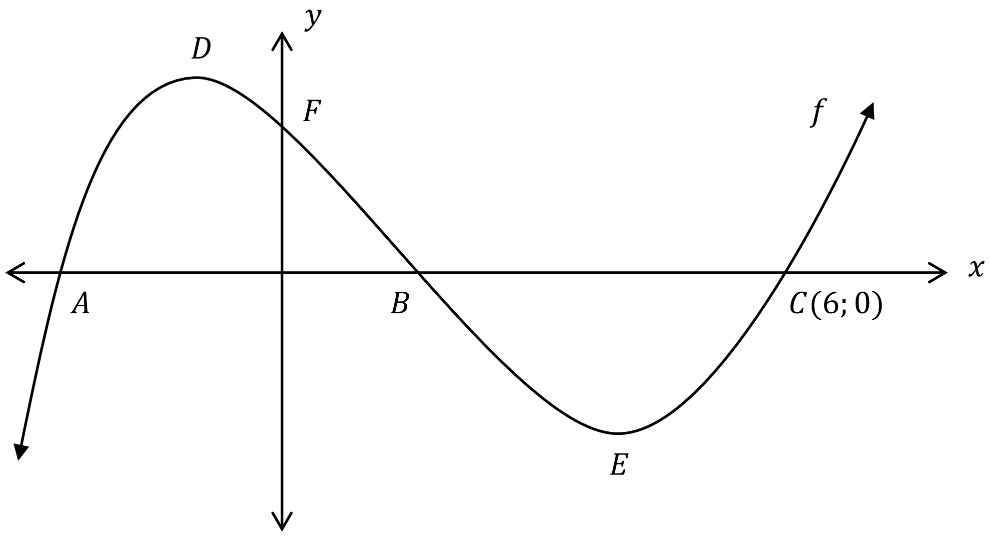
Calculate the value(s) of x where the gradient of f is equal to $\frac{8}{3}$. (5)

[14]

QUESTION 10

10.1 The sketch shows the graph of the function $f(x) = x^3 - 5x^2 - 8x + 12$.

A, B and $C(6;0)$ are the x-intercepts, F is the y-intercept and D and E are the stationary points of f.



10.1.1 Determine the coordinates of A and B. (4)

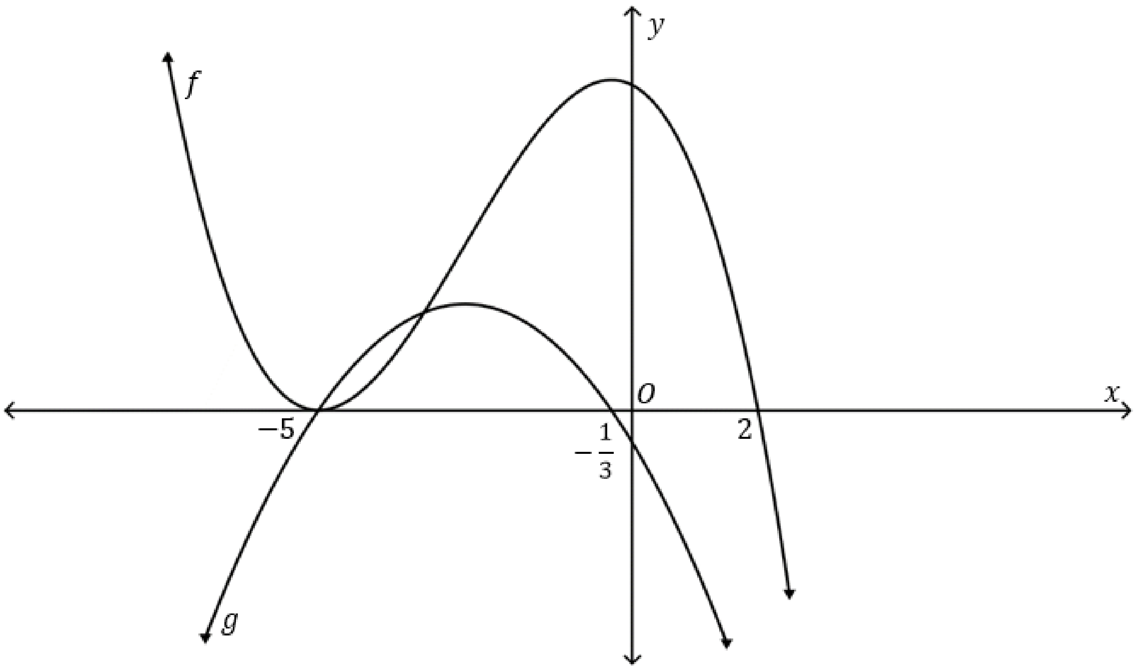
10.1.2 Determine the coordinates of D and E. (5)

10.2 The sketch shows the graphs of the following functions:

$$f(x) = ax^3 + bx^2 - 5x + 50$$

$$g(x) = f'(x) = px^2 + qx + t$$

The x-intercepts of f are $(-5; 0)$ and $(2; 0)$. The x-intercepts of g are $(-5; 0)$ and $\left(-\frac{1}{3}; 0\right)$.



Use the given information and sketch to answer the following questions:

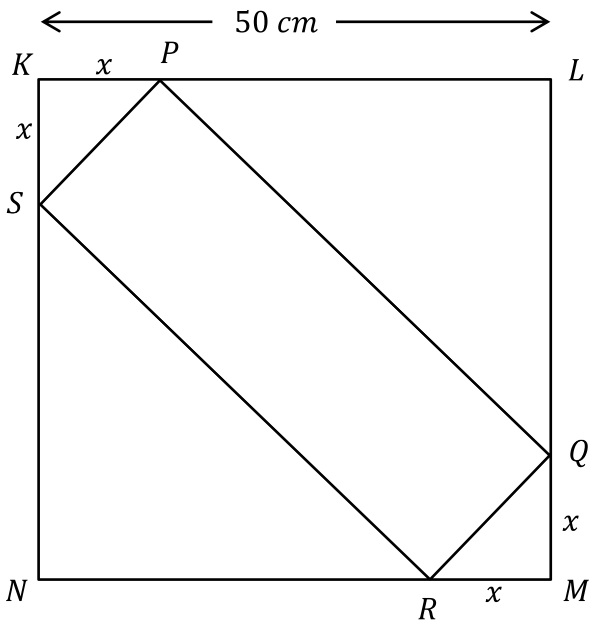
10.2.1 Determine the value of $g[f(2)]$. (2)

10.2.2 Calculate the average gradient of f over the interval $x \epsilon [-5; 0]$. (2)

10.2.3 Determine the values of x for which $f''(x) < 0$. (2)
[15]

QUESTION 11

$KLMN$ is a square in the figure below. $KL = 50\ cm$. Rectangle $PQRS$ is drawn inside square $KLMN$ so that $KP = KS = RM = QM = x$.



11.1 Show that the area (A) of rectangle $PQRS$ can be expressed as:

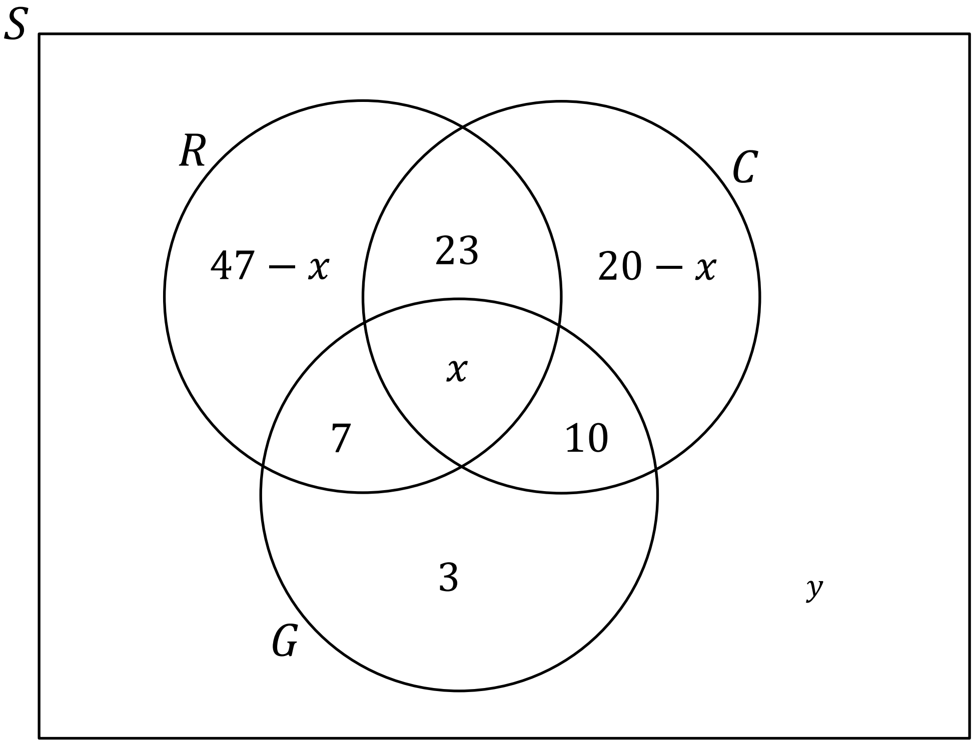
$$A = 100x - 2x^2$$ (4)

11.2 Hence determine the maximum area of $PQRS$. Show all calculations. (3)

[7]

QUESTION 12

12.1 There are 115 people in a group. The Venn-diagram below shows the number of people who enjoy listening to radio (R), enjoy gardening (G) and/or enjoy cooking (C). There are x people who enjoy all three activities. There are y people who do not enjoy any of the activities.



12.1.1 If there are 28 people who enjoy gardening, calculate the value of x. (1)

12.1.2 Hence determine the value of y. (1)

12.2 There are 5 loaves of brown bread (B) and 7 loaves of white bread (W) on the shelf at the local supermarket. Two clients, one followed by the other, each randomly selects a loaf of bread from the shelf and places it in their basket.

12.2.1 Determine the probability that the first client takes a loaf of white bread. (1)

12.2.2 Assume that the owner of the shop does not replace any loaves of bread on the shelf after a client has taken a loaf of bread.

Determine the probability that both clients take a loaf of brown bread. (3)

12.2.3 If the first client takes a loaf of white bread, the owner of the shop places a loaf of brown bread with the other loaves on the shelf. If the first client takes a loaf of brown bread, the owner of the shop places a loaf of white bread with the other loaves on the shelf.

Determine the probability that a loaf of white bread and a loaf of brown bread is sold to the two clients. (3)

12.3 The 9 letters in the word CELLPHONE are each written on a card and rearranged.

12.3.1 How many different arrangements can be made if the repeated letters (E and L) are considered as different? (1)

12.3.2 Determine the probability that the two E's will be placed next to each other if the repeated letters are considered as different. (3)

12.3.3 How many different arrangements that start with the letter P can be made if the repeated letters are considered as the same? (2)

[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$